

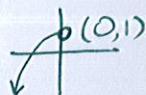
Describe the difference between the curves with parametric equations $x = -e^t$ and $x = \sin t$
 $y = 1 - e^{2t}$ and $y = \cos^2 t$. SCORE: ____ / 10 PTS

Discuss the rectangular equation(s) of the graphs, as well as the orientation and portion of the graph corresponding to the parametric equations.

BOTH CURVES CORRESPOND TO $y = 1 - x^2$

AS t GOES FROM $-\infty$ TO ∞

$x = -e^t$ GOES FROM ≈ 0 TO $-\infty$ BUT $x = \sin t$ OSCILLATES BETWEEN -1 AND



Use sigma notation to write the series $\frac{3!}{128} - \frac{4!}{192} + \frac{5!}{288} - \frac{6!}{432} + \dots - \frac{8!}{972}$. SCORE: ____ / 15 PTS

GEOMETRIC
 $r = -1.5$

$$\sum_{n=1}^6 \frac{(n+2)!}{32(-1.5)^{n-1}} \quad \text{or} \quad \sum_{n=3}^8 \frac{n!}{32(-1.5)^{n-3}} \quad \text{or} \quad \sum_{n=0}^5 \frac{(n+3)!}{32(-1.5)^n}$$

Using mathematical induction, prove that $\sum_{i=1}^n 3^{i+1} = \frac{3^{n+2} - 9}{2}$ for all positive integers n .

SCORE: ____ / 25 PTS

Do NOT use the finite geometric series formula in your proof.

BASIS STEP: WHEN $n=1$, $\sum_{i=1}^1 3^{i+1} = 3^2 = 9$

$$\frac{3^3 - 9}{2} = \frac{18}{2} = 9$$

INDUCTIVE STEP:

ASSUME $\sum_{i=1}^k 3^{i+1} = \frac{3^{k+2} - 9}{2}$ FOR SOME ARBITRARY BUT PARTICULAR INTEGER k

$$\begin{aligned} \sum_{i=1}^{k+1} 3^{i+1} &= \sum_{i=1}^k 3^{i+1} + 3^{k+2} \\ &= \frac{3^{k+2} - 9}{2} + 3^{k+2} \\ &= \frac{3^{k+2} - 9 + 2 \cdot 3^{k+2}}{2} \\ &= \frac{3 \cdot 3^{k+2} - 9}{2} \\ &= \frac{3^{k+3} - 9}{2} \\ &= \frac{3^{(k+1)+2} - 9}{2} \end{aligned}$$

BY MI,

$$\sum_{i=1}^n 3^{i+1} = \frac{3^{n+2} - 9}{2}$$

FOR ALL POSITIVE INTEGERS n

Simplify $\frac{(8n-2)!}{(8n+1)!}$.

SCORE: ____ / 10 PTS

$$\frac{(8n-2)!}{(8n+1)8n(8n-1)(8n-2)!} = \frac{1}{8n(8n+1)(8n-1)}$$

Eliminate the parameter to find rectangular equations corresponding to the parametric equations

$$x = \frac{t+1}{t-3}$$

$$y = \frac{t}{2-t}$$

SCORE: ____ / 15 PTS

For your final answer, write y as a simplified function of x .

$$x(t-3) = t+1$$

$$xt - 3x = t+1$$

$$xt - t = 3x+1$$

$$(x-1)t = 3x+1$$

$$t = \frac{3x+1}{x-1}$$

$$y = \frac{\frac{3x+1}{x-1}}{2 - \frac{3x+1}{x-1}}$$

$$= \frac{3x+1}{2(x-1) - (3x+1)}$$

$$= \frac{3x+1}{-x-3} = -\frac{3x+1}{x+3}$$

Consider the expression $(19x^7 - 13x^4)^{21}$.

SCORE: ____ / 25 PTS

[a] Write the expansion of the expression using sigma notation. **Your answer may use ! but not ${}_nC_r$ notation.**

Simplify all exponents.

$$\sum_{r=0}^{21} {}_{21}C_r (19x^7)^{21-r} (-13x^4)^r = \sum_{r=0}^{21} \frac{21!}{r!(21-r)!} 19^{21-r} (-13)^r x^{7(21-r)+4r}$$

$$= \sum_{r=0}^{21} \frac{21!}{r!(21-r)!} 19^{21-r} (-13)^r x^{147-3r}$$

[b] Find the coefficient of x^{108} in the expansion. **Your answer may use ! but not ${}_nC_r$ notation.**

$$147-3r = 108$$

$$-3r = -39$$

$$r = 13$$

$$\frac{21!}{13!8!} 19^8 (-13)^{13} = -\frac{21!}{13!8!} 19^8 13^{13}$$

GJ spent 51 hours sleeping the first week of the quarter. Each week afterwards, GJ's sleep time was 3% less than the previous week. If the quarter was 11 weeks long, how much time did GJ sleep over the entire quarter?

SCORE: ____ / 10 PTS

$$51 + 51(0.97) + 51(0.97)^2 + \dots + 51(0.97)^{10} \quad \text{GEOMETRIC, } r=0.97$$

$$= \frac{51(1-0.97^{11})}{1-0.97} = 483.99 \text{ HOURS}$$

Find the sum of the series $232 + 225 + 218 + 211 + 204 + \dots - 377$.

SCORE: ____ / 15 PTS

ARITHMETIC, $d = -7$

$$232 - 7(n-1) = -377$$

$$-7(n-1) = -609$$

$$n-1 = 87$$

$$n = 88$$

$$S_{88} = \frac{1}{2}(88)(232 + -377) = -6380$$

HJ is standing 20 feet from IJ, who is 4 feet tall. HJ throws a football at 25 feet per second in IJ's direction, at an angle of 61.93° with the horizontal, from an initial height of 6 feet.

SCORE: ____ / 25 PTS

NOTE: $\sin 61.93^\circ = \frac{15}{17}$ and $\cos 61.93^\circ = \frac{8}{17}$

[a] Write parametric equations for the position of the football.

$$x = (25 \cos 61.93^\circ)t = \frac{200}{17}t$$

$$y = 6 + (25 \sin 61.93^\circ)t - 16t^2 = 6 + \frac{375}{17}t - 16t^2$$

[b] Does the football hit IJ, go over IJ's head, or hit the ground before reaching IJ?

$$x = 20$$

$$\frac{200}{17}t = 20$$

$$t = 1.7$$

$$y = 6 + \frac{375}{17}(1.7) - 16(1.7)^2$$

$$y = -2.74 < 0$$

THE FOOTBALL HITS THE GROUND
BEFORE REACHING IJ